STAT 237: HW3

DUE ELECTRONICALLY VIA THE RSTUDIO SERVER FRIDAY 03/18/22 BY 5PM

1. Let μ represent the probability that a coin with unknown properties comes up heads.

   (i) Suppose we use a $\mathsf{Beta}(4, 4)$ prior on μ. Qualtitatively, **what does this choice say** about what we think is plausible?

   (ii) Suppose we flip the coin once and get heads. What is the **posterior distribution** for μ?

   (iii) Suppose we flip the same coin again and get heads again. **Find the new posterior in two ways**: First, use **iterative updating**, using the posterior from after the first flip as the new prior. Then, use **batch updating**, starting with the original prior and updating based on the likelihood of two heads in a row.

   (iv) Suppose a third flip is tails. **What is the posterior distribution now?** Use whichever update method you prefer.

   (v) Suppose we had instead seen tails first, followed by the two heads. **Find the posterior distribution for this sequence** of results.

   (vi) If instead of the $\mathsf{Beta}(4, 4)$ distribution we had instead used a uniform distribution as the prior, **how many flips would we need to observe before arriving at the same posterior** we did here? How many of these would need to have been heads?

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2. An upcoming U.S. election features two candidates: a Democrat and a Republican. Let μ represent the proportion of the general population of a particular region who prefer the Democratic candidate.

   (a) Suppose the region is the whole country, and you have a strong prior belief that the population will be fairly evenly divided. If you were to encode this belief as a Beta distribution prior for μ, **what parameters would you select? Explain** your choice (There is of course no unique right answer here)

   (b) Suppose the region is a particular county, but you don't know anything about the demographics or location of that county. Since counties in the U.S. are frequently somewhat politically homogeneous, you have a prior belief that the population will *not* be particularly evenly divided, but it could lean either way. If you were to encode this belief as a Beta distribution prior for μ, **what parameters would you select? Explain** your choice (There is no unique right answer here either!)

   (c) Using each of the priors you chose, find the **marginal likelihood** that a person chosen at random from the region in question prefers the Democratic candidate. That is, if $X \mid \mu$ has a Bernoulli distribution with parameter μ, find the marginal probability $p_X(1)$.

   (d) Suppose a polling firm has preliminary data from just 10 people (so far). Of the 10 people polled (each of whom was selected at random from the region in question), 8 of them preferred the Democrat. For each of the priors you chose, find the **posterior distribution** of μ, and use it to find the **posterior predictive probability** that the next person asked will prefer the Democrat. That is, find $p(x_{11} \mid x_1, \ldots, x_{10})$.