STAT 237: HW2

DUE ELECTRONICALLY VIA THE RSTUDIO SERVER FRIDAY 03/11/22 BY 5PM

Suppose we have a six-sided die from a board game. The sides each have a number of "pips" (dots), with one side having 1, another side having 2, etc. Let X be a random variable representing the number of pips facing up on the die when we roll it (in other words, the result of the roll). Then, $X = x$ represents the event where the result of the roll is x.

1. What is the **range** of X?
2. Suppose hypothesis A (H_A) says the die is fair so that each face has an equal chance of turning up. Write out an expression for the PMF of X conditioned on H_A, $p(x \mid H_A)$, and then find $\mathbb{E}[X \mid H_A]$.
3. On most six-sided dice, the faces are arranged so that opposite faces sum to 7. Suppose the die has this arrangement of faces. Hypothesis B (H_B) says that sides with more pips are more likely to land facing **down**, such that the probability that a face with y pips lands **down** is $c \times y$ for some constant c. What must c be? (Hint: Remember that the probabilities of an exhaustive set of mutually exclusive outcomes must sum to 1) Find an expression for $p(x \mid H_B)$ and then find $\mathbb{E}[X \mid H_B]$
4. In reality there are *many* other ways a die could be unbalanced, but for simplicity imagine that H_A and H_B are the only possibilities. Suppose you start with the **prior** that $p(H_A) = 0.80$ and $p(H_B) = 0.20$. What outcome x would provide the biggest boost to the plausibility of H_B? Call this outcome x_*. Find $p(H_B \mid x_*)$.
5. Suppose the first roll is x_*. We can use $p(H_B \mid x_*)$ as a prior probability of H_B going into the second roll. Are there any outcomes of the second roll that would cause the posterior probabilty of H_B after both rolls to rise above 50%? Explain and show any necessary calculations to justify your answer.

Date: Last Revised March 7, 2022.