STAT 215
Confidence Intervals

Colin Reimer Dawson

Oberlin College

Sept. 21, 2017

Outline

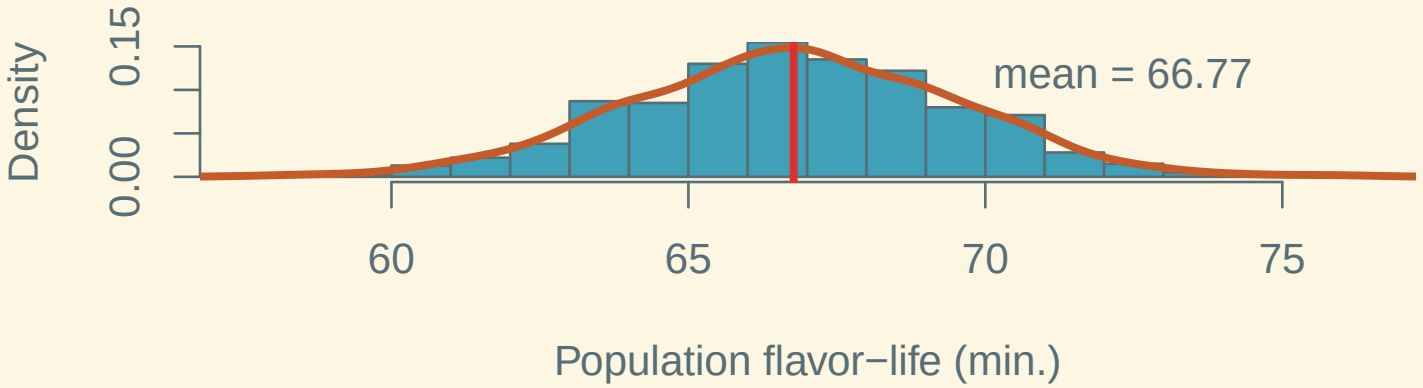
Two Main Goals of Inference

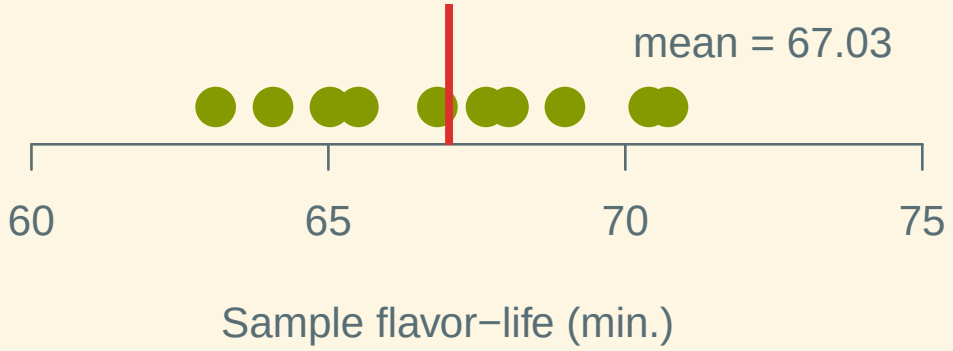
1. Assessing strength of evidence about "yes/no" questions (hypothesis testing)
2. Estimating unknown quantities in a population using a sample (confidence intervals)

Outline

Variability due to Sampling

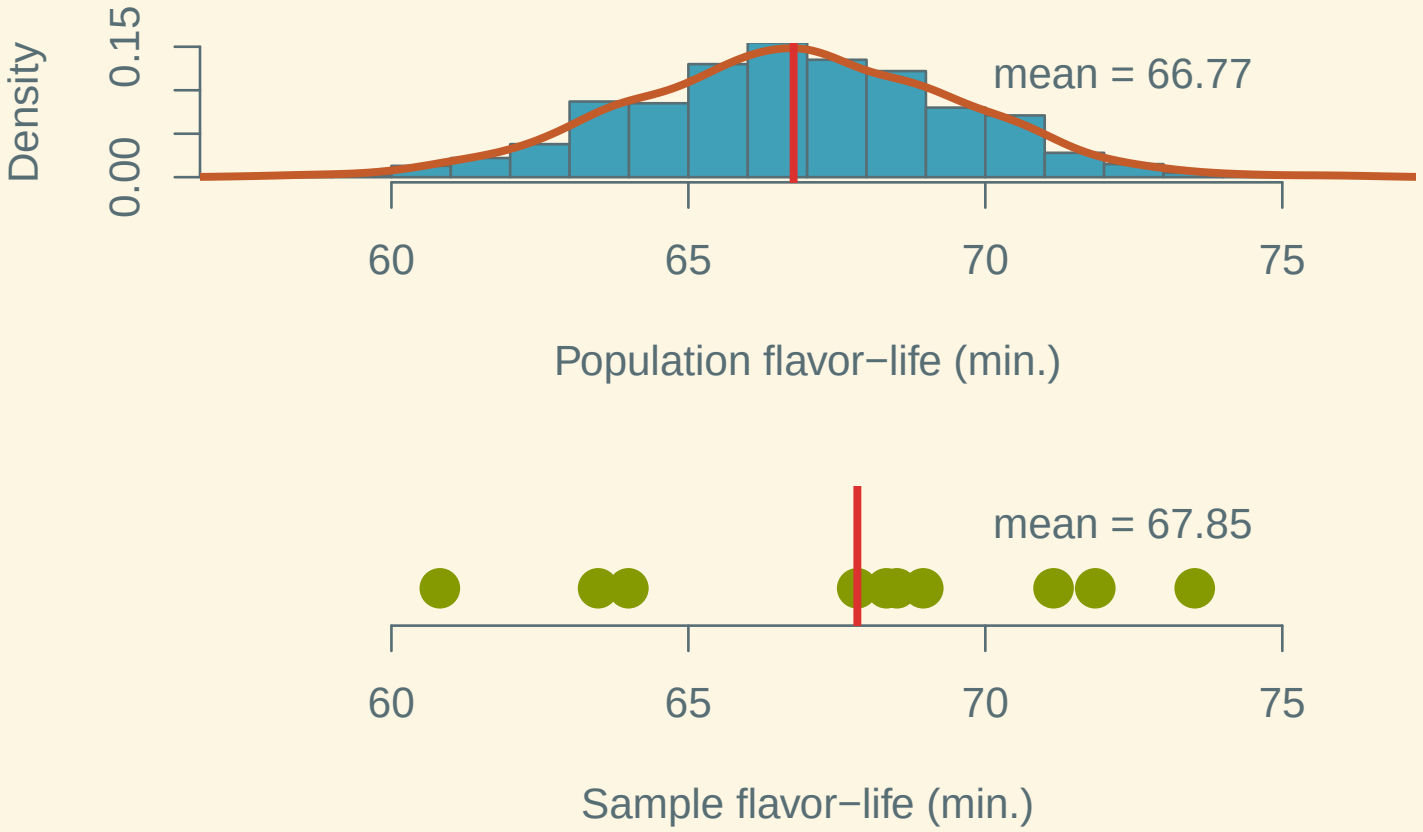
If we take a random sample from this population, it might look like this





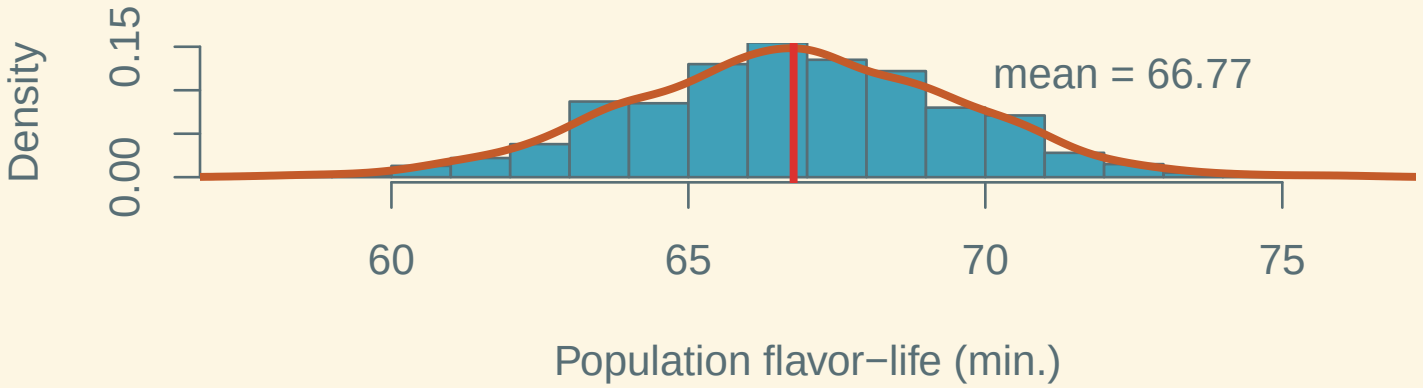
Variability due to Sampling

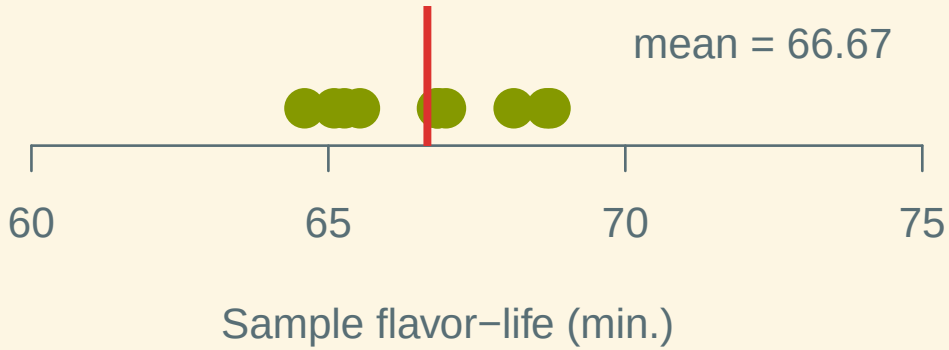
Or this



Variability due to Sampling

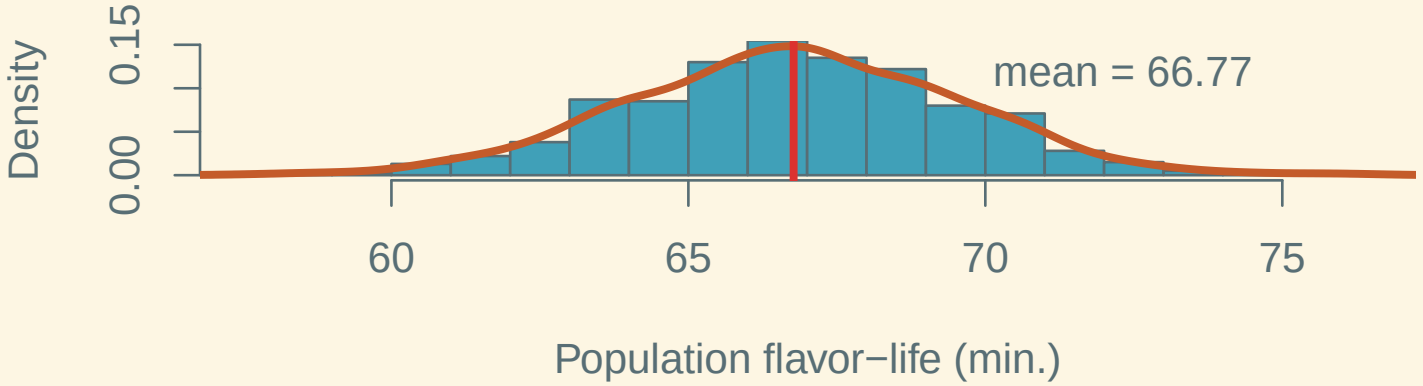
Or this

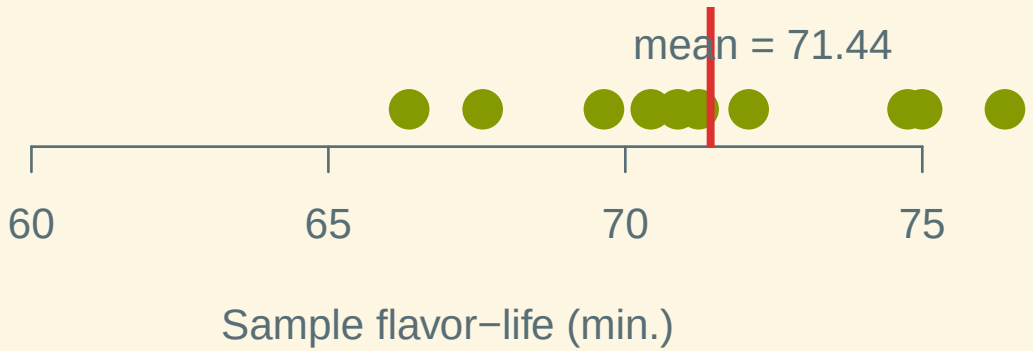




Variability due to Sampling

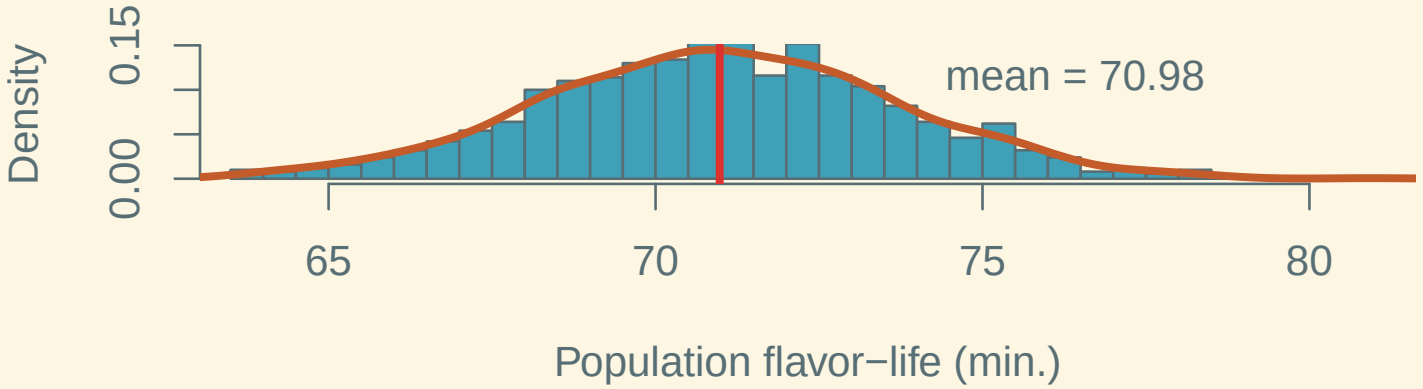
This one could happen, but it's not very likely.

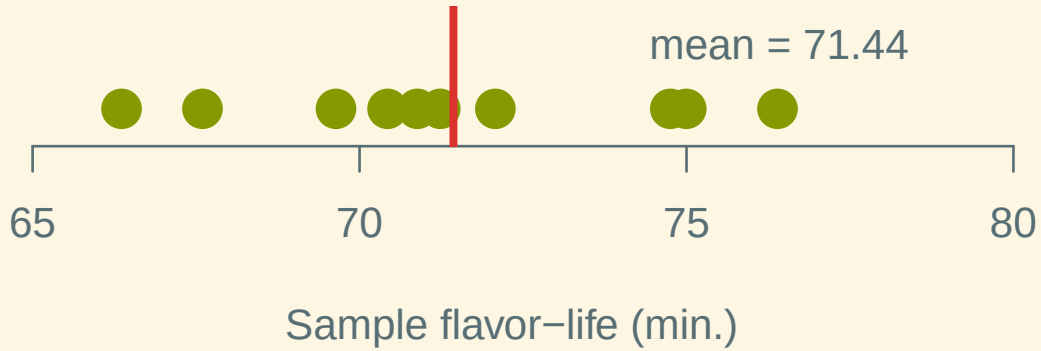




Variability due to Sampling

But if the population looked like this instead...





then the first three samples are unlikely, whereas the last one is more likely.

Variability due to Sampling

- Each sample differs from the population, so sample information is an imperfect reflection.
- However, there is information about the population, since some populations are more likely than others to produce the given sample.
- If we imagine a continuum of populations (or just population means), some are more plausible than others *because they make the data more likely*.

Sampling Distributions

Sampling Distribution Definition

Consider all possible random samples of a fixed size, n from a population. Each one has its own value for a particular **statistic** (like $\bar{x}$). A **sampling distribution** is the collection of all of of those $\bar{x}$ values (or whatever the statistic is)

Self-Check

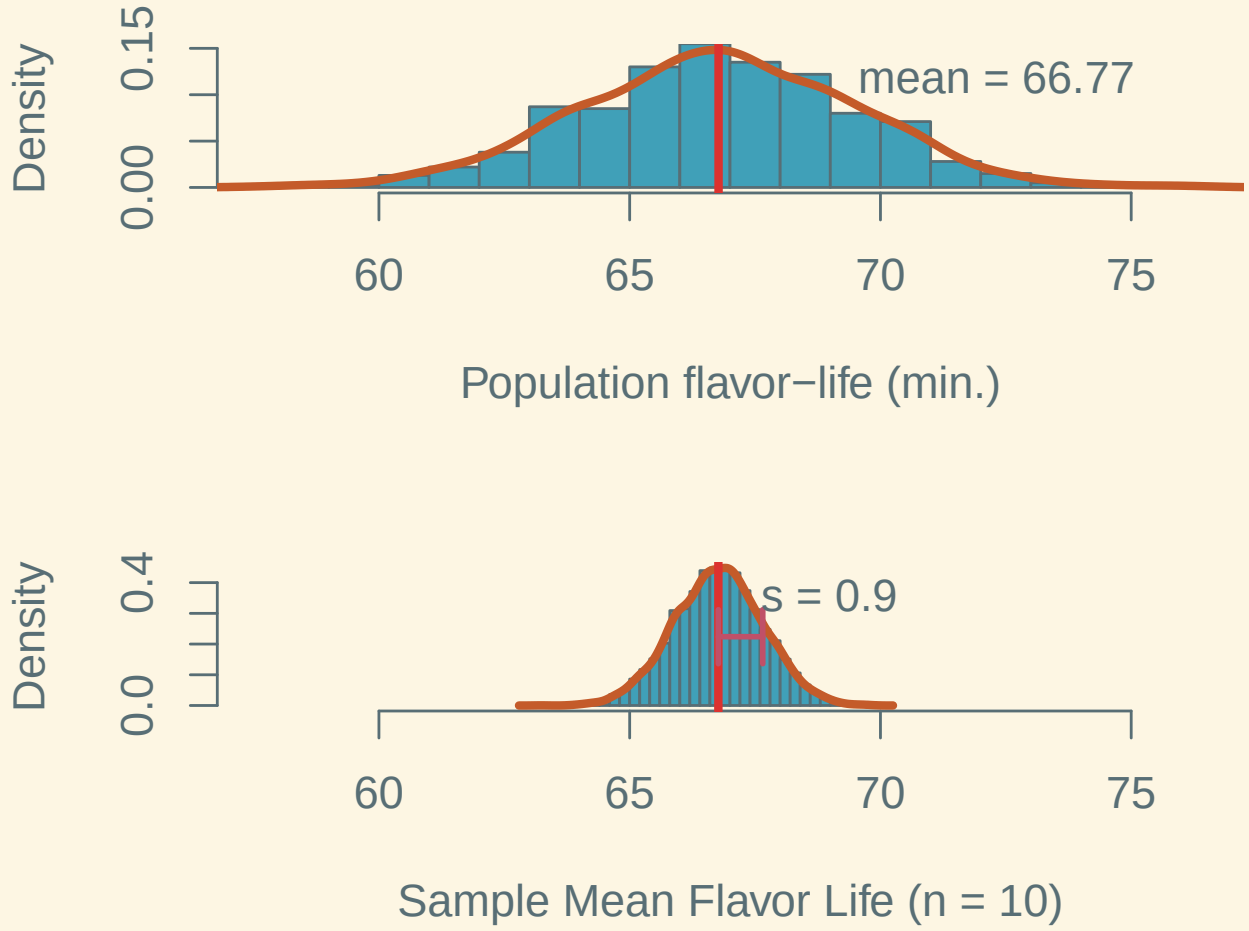
1. What are the cases in the context of a sampling distribution?
   Possible samples of a fixed size n
2. What is the variable in the relevant sampling distribution for the gumball life example? Each case has its own **sample mean**

Standard Error

Standard Error Definition

The distribution of a quantitative variable has a standard deviation. The distribution of a quantitative *sample statistic* (like a mean) has a standard deviation too. This has a special name: the **standard error** (e.g., "of the mean").

Sampling Distribution of Gumball Means



Properties of Sampling Distribution

Most (about 95%) of *simple random* samples have a sample mean ($\bar{x}$) which is within 2 Standard Errors of the population mean (μ).

Outline

Margins of Error

In a recent ABC/Washington Post poll, a sample of 821 registered voters was asked who they would vote for in a hypothetical general election matchup between Hillary Clinton and Donald Trump. 46% of respondents picked Clinton and 43% picked Trump. **The poll's margin of error was 3 percentage points.**

- What's the meaning of that 3%?

Margin of Error

A 95% margin of error of 3 points means that 95% of surveys with the same procedure and sample size will yield sample statistics which are within 3 points of the corresponding population parameter.

Pair Quiz: Measures of "Error"

Most (about 95%) of *simple random* samples have a sample mean ($\bar{x}$) which is within 2 Standard Errors of the population mean (μ).

Margin of Error

A 95% margin of error of 3 points means that 95% of surveys with the same procedure and sample size will yield sample statistics which are within 3 points of the corresponding population parameter.

In many cases a 95% Margin of Error is about 2 Standard Errors.

Confidence Intervals

- A *point estimate* of some population parameter (like a mean), together with some measure of our confidence/uncertainty (e.g., MoE), defines a **confidence interval**.
- "With 95% confidence, the mean flavor-life of our gumballs is between 65.3 and 67.1 minutes."
- "With 95% confidence, between 43 (i.e, $46 - 3$) and 49 (i.e., $46 + 3$) percent of registered voters prefer Hillary Clinton to Donald Trump."
- "With 95% confidence, between 40 ($43 - 3$) and 46 ($43 + 3$) percent of registered voters prefer Donald Trump to Hillary Clinton."

Self-Check: Confidence Intervals

HBO Sports/Marist gave 1253 U.S. adults the following poll question in spring 2015 (I have edited for length):

> *"Top college men's football and basketball programs bring in a lot of money to their schools... Do you think student athletes in [these top programs] should be paid for the hours they are required to spend practicing, traveling, and playing on the team, OR should not be paid given the value of their scholarship and a chance to earn a degree?"*

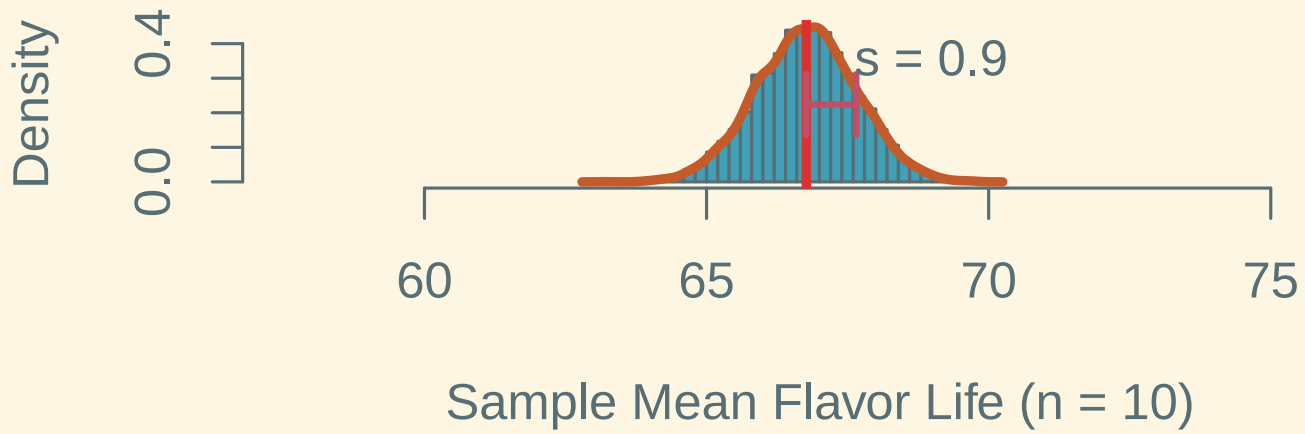
This poll's 95% margin of error is 2.8%. The results are given in the following table. Find a 95% confidence interval for the percentage of U.S. adults who chose the first option.

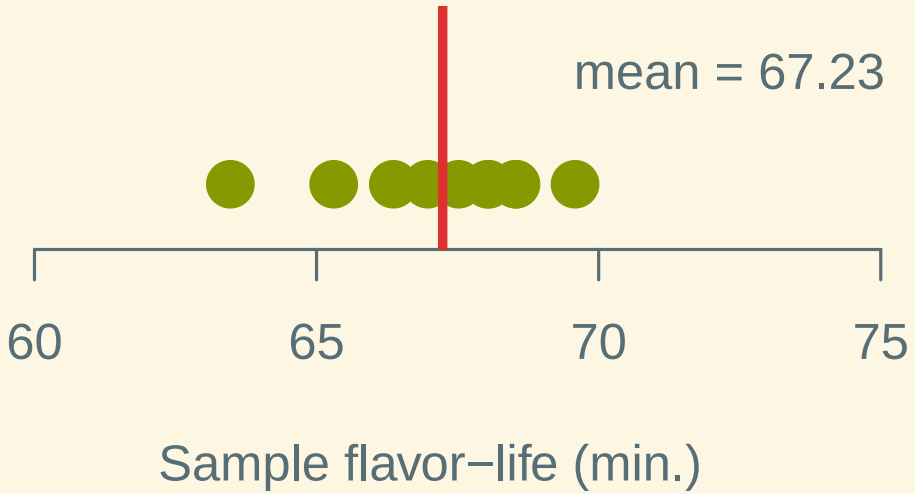
Should be paid	Should not be paid	Unsure
33%	65%	2%

True or False?

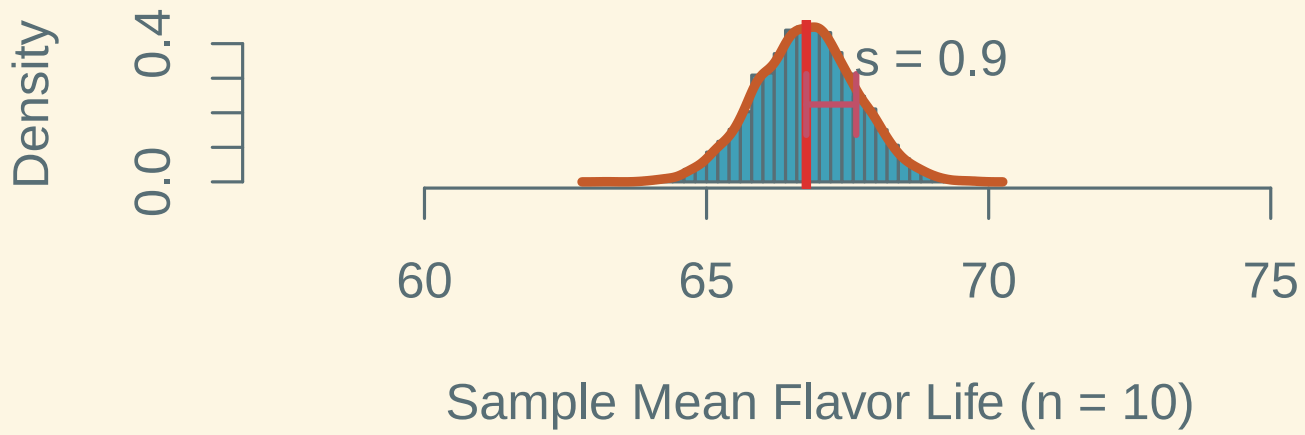
- 95% CIs contain 95% of the population values. False. They represent uncertainty about a population parameter, not about individual points.
- 95% of sample means fall in the 95% CI for the mean. False. Any given CI is centered around the sample mean for that sample, so the sample mean is inside 100% of the time.
- 95% of samples produce confidence intervals that contain the population parameter. True! This is the definition of a CI.

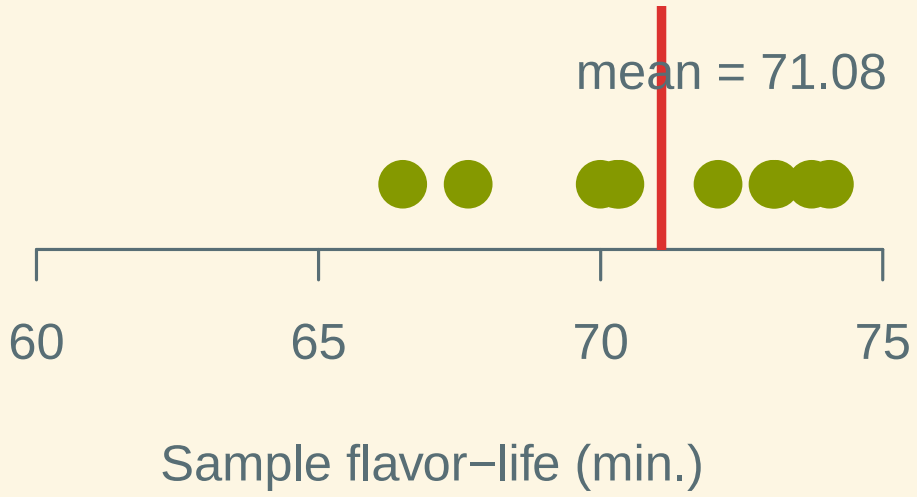
Confidence Interval of Gumball Flavor Life





A Different Confidence Interval of Gumball Flavor Life





Ok, but...

In reality we only have one sample. How do we know what the standard error is?

- Standard error depends on population characteristics, particularly variability
- We can use the sample to estimate not only the parameter of interest (e.g., mean, proportion), but also the variability.
- Two approaches: (1) Simulation, (2) Probability theory