STAT 213
Polynomials and Interactions

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Notes

Outline

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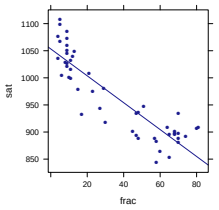
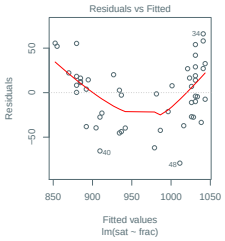
Example: State SAT Scores

```
library("mosaicData"); data("SAT")      ## sat = mean SAT score per state
slr.model <- lm(sat ~ frac, data = SAT) ## frac = % taking SAT
```

```
plotModel(slr.model)
```




```
plot(slr.model, which = 1)
```



Notes

Polynomial Regression

We can create "new" predictors from old, e.g.:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \cdots + \beta_p X^p$$

$$p = \begin{cases} 1, & \text{linear} \\ 2, & \text{quadratic} \\ 3, & \text{cubic} \\ etc. & \end{cases}$$

Notes

R: Three Equivalent Methods

Method 1: Explicit Variable Creation

```
SAT.augmented <- mutate(SAT, frac.squared = frac^2)
quadratic.model <- lm(sat ~ frac + frac.squared, data = SAT.augmented)
```

Method 2: Inline transformation (note use of `I()`)

```
quadratic.model <- lm(sat ~ frac + I(frac^2), data = SAT.augmented)
```

Method 3: Using `poly()` to generate polynomials

```
quadratic.model <- lm(sat ~ poly(frac, degree = 2, raw = TRUE),
                      data = SAT.augmented)
```

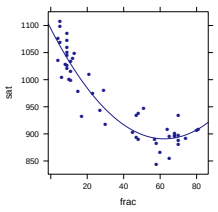
```
Call:
lm(formula = sat ~ frac + I(frac^2), data = SAT.augmented)

Coefficients:
(Intercept)         frac    I(frac^2)
 1094.09787    -6.52850     0.05242
```

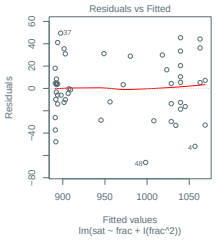

Notes

Example: State SAT Scores

```
## This doesn't work w/ "Method 1"
plotModel(quadratic.model)
```

```
plot(quadratic.model, which = 1)
```





Notes

ASSESS: Do we need the quadratic term?

```
summary(quadratic.model) %>% coefficients() %>% round(digits = 2)
```

```
               Estimate Std. Error t value Pr(>|t|)
(Intercept)  1094.10       9.64  113.45        0
frac           -6.53       0.73   -8.94        0
I(frac^2)       0.05       0.01    5.65        0
```


Notes

Selecting Polynomial Order

- Start with a higher-order model, then remove highest order term if not significant.
- Repeat until highest order term is significant.
- To be safe(ish): nested F-test between final model and highest-order model.
- Don't remove lower order terms even if nonsignificant!

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Interaction Terms and Second-Order Models

Consider the model:

$$\texttt{sat} = \beta_0 + \beta_1 \cdot \texttt{frac} + \beta_2 \cdot \texttt{expend} + \beta_3 \cdot \texttt{frac} \cdot \texttt{expend} + \varepsilon$$

where expend is state education expenditure per pupil.

How can we interpret β_3? Represents change in slope relating sat to expend for *each unit increase* in frac (or vice versa)

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Interaction Visualization

Demo

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The Economic Value of a College Degree

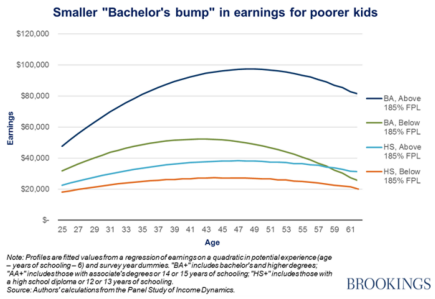

Figure: Source: `http://www.pbs.org/newshour/making-sense/if-you-grew-up-poor-your-college-degree-may-be-worth-less/`

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