STAT 213
Multiple Regression I

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Outline

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Quantitative Vs. Categorical Predictor and Response

		Response	
		Quantitative	**Categorical**
Predictor(s)	**Quantitative**	SLR	Logistic Reg.
	Categorical	ANOVA	
	Multiple	Multiple Reg.	

The Multiple Regression Model

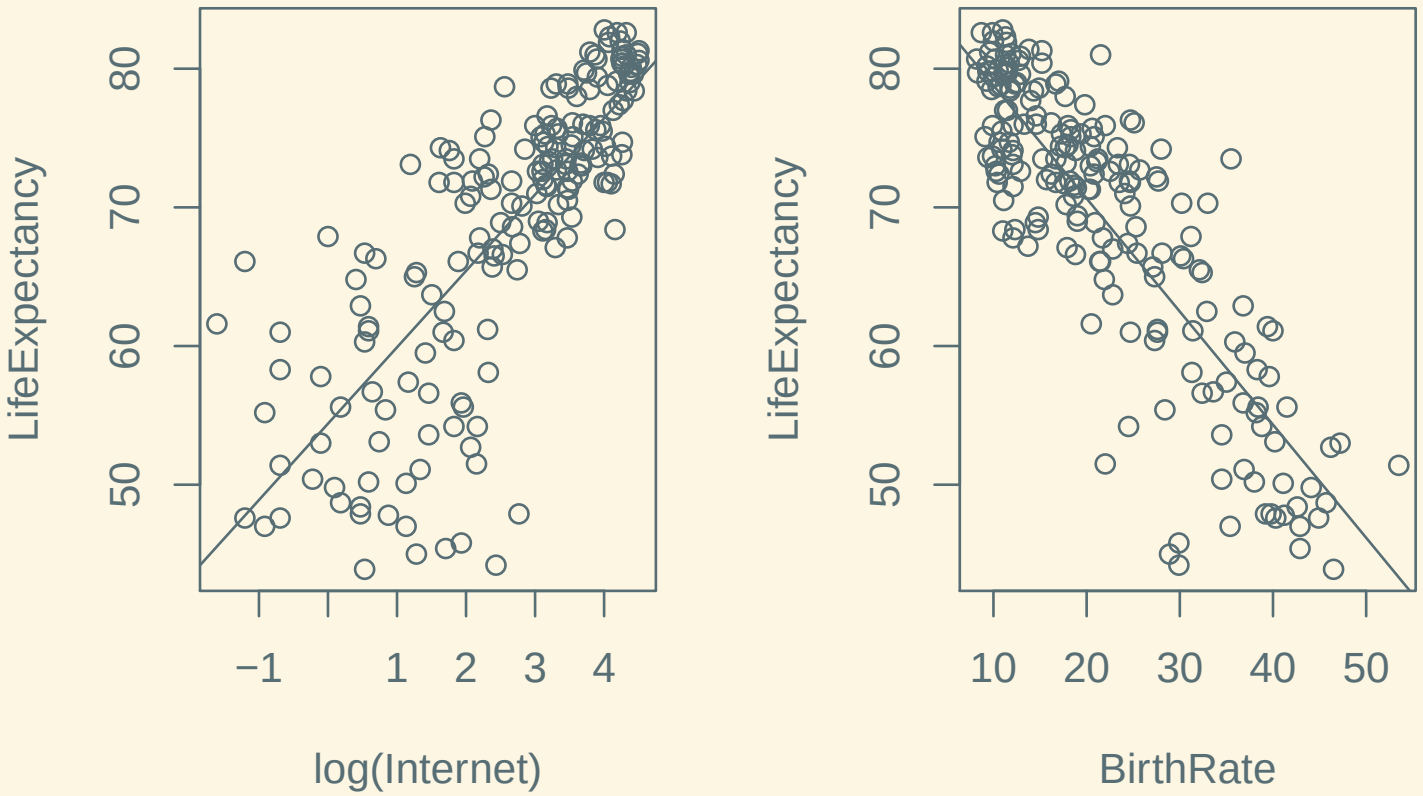
DATA = PATTERN + IDIOSYNCRACIES

The Multiple Regression "Population" Model

$$Y_i = f(X_{i1}, \ldots, X_{iK}) + \varepsilon_i$$
$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_K X_{iK} + \varepsilon_i$$

where i picks out cases, k picks out variables. X_{ik} is the kth predictor for case i. One β_k for each predictor X_k

Example: Life Expectancy



We could fit separate regression models for each predictor...

SLR Model Using log(Internet)

```
model1 <- lm(LifeExpectancy ~ log(Internet), data = AllCountries)
summary(model1) %>% coefficients() %>% round(digits = 2)
```

```
                Estimate Std. Error t value Pr(>|t|)
(Intercept)        54.40       0.95   57.42        0
log(Internet)       5.51       0.32   17.47        0
```

$$\widehat{\texttt{LifeExpectancy}}_i = 54.40 + 5.51 \cdot \log(\texttt{Internet}_i)$$

and

$$\texttt{LifeExpectancy}_i = 54.40 + 5.51 \cdot \log(\texttt{Internet}_i) + \varepsilon_i$$

SLR Model Using BirthRate

```
model2 <- lm(LifeExpectancy ~ BirthRate, data = AllCountries)
summary(model2) %>% coefficients() %>% round(digits = 2)

             Estimate Std. Error t value Pr(>|t|)
(Intercept)     86.95       0.88   98.43        0
BirthRate       -0.82       0.04  -22.64        0
```

$$\widehat{\texttt{LifeExpectancy}}_i = 86.95 - 0.82 \cdot \texttt{BirthRate}_i$$

and

$$\texttt{LifeExpectancy}_i = 86.95 - 0.82 \cdot \texttt{BirthRate}_i + \varepsilon_i$$

Or Fit Both at Once

```
model3 <- lm(LifeExpectancy ~ log(Internet) + BirthRate, data = AllCountries)
summary(model3) %>% coefficients() %>% round(digits = 2)
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	77.23	2.33	33.15	0
log(Internet)	1.95	0.43	4.59	0
BirthRate	-0.61	0.06	-10.36	0

$$\widehat{\texttt{LifeExpectancy}}_i = 77.23 + 1.95 \log(\texttt{Internet}_i) - 0.61 \cdot \texttt{BirthRate}_i$$

and

$$\texttt{LifeExpectancy}_i = 77.23 + 1.95 \log(\texttt{Internet}_i) - 0.61 \cdot \texttt{BirthRate}_i + \varepsilon_i$$

Single Multiple Regression vs. Multiple Single Regressions

- What is the added value in using both predictors together, vs. fitting each separately?
- There are disadvantages:
  - Harder to interpret
  - Can't easily plot
- Advantages...
  - If we actually know both predictors, we get a single prediction, instead of two conflicting ones
  - Can "control for" one predictor and test the other

What is a Case?

- Q: What does a single case consist of for a multiple regression model?
- A: A complete case has a value for Y, and for *each* X.

The Four-Step Process: Multiple Regression

1. CHOOSE a form of the model
   - Select predictors
   - Choose any transformations of predictors
2. FIT: Estimate
   - coefficients: $\hat{\beta}_1, \hat{\beta}_1, \ldots, \hat{\beta}_k$
   - residual variance $\hat{\sigma}^2_\varepsilon$
3. ASSESS the fit
   - Examine residuals (may need to return to step 1)
   - Test individual predictors (t-tests)
   - Test/measure overall fit (ANOVA, R^2)
   - Model comparison/selection
4. USE the model
   - Make predictions
   - Construct CIs and PIs

Outline

CHOOSE: Active Pulse Rate

```
library(Stat2Data); data("Pulse")
head(Pulse, n = 3)
```

```
  Active Rest Smoke Gender Exercise Hgt Wgt
1     97   78     0      1        1  63 119
2     82   68     1      0        3  70 225
3     88   62     0      0        3  72 175
```

$$\texttt{Active}_i = \beta_0 + \beta_1 \cdot \texttt{Rest}_i + \beta_2 \cdot \texttt{Hgt}_i + \beta_3 \cdot \texttt{Wgt}_i + \varepsilon_i$$

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FIT: Estimate Coefficients

The Multiple Regression Population Model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_K X_{iK} + \varepsilon_i$$

The Multiple Regression Fitted Model

$$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_{i1} + \cdots + \hat{\beta}_K X_{1K} + \hat{\varepsilon}_i$$

How to choose $\hat{\beta}_k$s? Minimize SSE! (Requires linear algebra / vector calculus)

FIT: Estimate Coefficients

```
my.model <- lm(Active ~ Rest + Hgt + Wgt, data = Pulse)
coef(my.model) %>% round(digits = 1)

(Intercept)        Rest         Hgt         Wgt
       57.3         1.1        -0.9         0.1
```

$$\texttt{Active}_i = 57.3 + 1.1 \cdot \texttt{Rest}_i - 0.9 \cdot \texttt{Hgt}_i + 0.1 \cdot \texttt{Wgt}_i + \varepsilon_i$$

FIT: Estimate Residual Variance

Recall Variance Decomposition for Regression:

$$\sum_i (Y_i - \bar{Y})^2 = \sum_i (\hat{Y}_i - \bar{Y})^2 + \sum_i (Y_i - \hat{Y}_i)^2$$

$$SS_{Total} = SS_{Model} + SS_{Error}$$

Recall ANOVA Table:

$$MS_{Model} = SS_{Model}/df_{Model}$$
$$MS_{Error} = SS_{Error}/df_{Error}$$

where MS_{Error} represents $\hat{\sigma}^2_\varepsilon$. So... what are df_{Model} and df_{Error}?

Regression Degrees of Freedom

$df_{Model} = K$ where K is the number of predictors

This is the number of extra "free parameters" (compared to the null model)

$df_{Error} = N - K - 1$ where N is the sample size

This is the number of "pieces of information" we have about the sizes of the residuals. (Can fit any N points exactly with N coefficients including the intercept.)

FIT: Estimate Residual Variance

$$\hat{\sigma}^2_{\varepsilon} = MS_{Error} = \frac{SS_{Error}}{df_{Error}} = \frac{\sum_{i=1}^{N}(Y_i - \hat{Y}_i)^2}{N - K - 1}$$

FIT: Estimate Residual Variance

```
summary(my.model)


    Call:
    lm(formula = Active ~ Rest + Hgt + Wgt, data = Pulse)

    Residuals:
        Min      1Q  Median      3Q     Max
    -35.245  -9.968  -2.458   6.697  64.716

    Coefficients:
                Estimate Std. Error t value Pr(>|t|)
    (Intercept) 57.25991   25.01347   2.289   0.0230 *
    Rest         1.12647    0.10162  11.086   <2e-16 ***
    Hgt         -0.88060    0.40535  -2.172   0.0309 *
    Wgt          0.10855    0.04699   2.310   0.0218 *
    ---
    Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

    Residual standard error: 14.91 on 228 degrees of freedom
    Multiple R-squared:  0.3808,Adjusted R-squared:  0.3726
    F-statistic: 46.73 on 3 and 228 DF,  p-value: < 2.2e-16
```

FIT: The Final Model

$$\texttt{Active}_i = 57.3 + 1.1 \cdot \texttt{Rest}_i - 0.9 \cdot \texttt{Hgt}_i + 0.1 \cdot \texttt{Wgt} + \varepsilon_i$$

where $\varepsilon_i \sim \mathcal{N}(0, 14.9)$

Next

- ASSESSing MLR models
- Binary Predictors