STAT 113
Normal-Based Inference for Differences of Proportions

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Cases to Address

We will develop standard error formulas to do CIs and tests for the following parameters:

1. Single Proportion (last year)
2. Difference of Proportions (today)
3. Single Mean (Wednesday)
4. Difference of Means (Wednesday)
5. Mean of Differences (Lab this Thursday)

Reference Table: Sampling Distributions

Param.	Stat.	$\widehat{SE}$	Distribution	Degrees of Freedom
p	$\hat{p}$	$\sqrt{\frac{p(1-p)}{n}}$	$\mathcal{N}(0,1)$	
$p_A - p_B$	$\hat{p}_A - \hat{p}_B$	$\sqrt{\widehat{SE}_{\hat{p}_A}^2 + \widehat{SE}_{\hat{p}_B}^2}$	$\mathcal{N}(0,1)$	
μ	$\bar{x}$	$\sqrt{\frac{s^2}{n}}$	t	$n-1$
$\mu_A - \mu_B$	$\bar{x}_A - \bar{x}_B$	$\sqrt{\widehat{SE}_{\bar{x}_A}^2 + \widehat{SE}_{\bar{x}_B}^2}$	t	$\min(n_A, n_B) - 1$
μ_{diff}	$\bar{x}_{\text{diff}}$	$\sqrt{\frac{s_{\text{diff}}^2}{n_{\text{diff}}}}$	t	$n_{\text{diff}} - 1$
ρ	r	$\sqrt{\frac{1-r^2}{n-2}}$	t	$n-2$

$$CI: \quad \text{Observed Statistic} \pm z_{\text{endpoint}} \cdot \widehat{SE}$$

$$\text{Test Statistic}: \quad \frac{\text{Observed Statistic} - \text{Null Param.}}{\widehat{SE}}$$

Standard Error of a Difference

CI for Difference of Proportions

Hypothesis Test for Difference of Proportions

Example: Penguins Again! But Different!

Penguin Breeding

- The scientists who studied whether metal bands were harmful to penguin survival also examined whether they affected the penguins' breeding patterns.
- For the metal-band group, **39 of 122 (32%) penguin-seasons resulted in offspring**
- The controls had offpsring in **70 of 160 penguin-seasons (44%)**.

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- If we are focused on **the effect of metal bands on penguins' reproductive "success"**, what **parameter** would we want to focus on?

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- The controls had offpsring in **70 of 160 penguin-seasons (44%)**.

- If we are focused on **the effect of metal bands on penguins' reproductive "success"**, what **parameter** would we want to focus on?
- What is the corresponding **statistic**?

Outline

Standard Error of a Difference

CI for Difference of Proportions

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Variance and Standard Error of Differences

- The **difference** between two statistics is affected by **both statistics**.

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- If the errors are in **opposite directions**, the errors would **add**.

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- The **difference** between two statistics is affected by **both statistics**.
- Each statistic differs from its corresponding parameter: it has some **sampling error**.
- If the two sources of sampling error are in the **same direction**, error would **cancel out** when we looked at the **difference**.
- If the errors are in **opposite directions**, the errors would **add**.
- What can we expect the "average" result to be?

"Pythagorean Theorem" of Standard Errors

When two sample statistics, come from **sets of unrelated cases**, the estimation error associated with their difference obeys a **Pythagorean Theorem**:

$$SE^2_{\text{difference}} = SE^2_{\text{first}} + SE^2_{\text{second}}$$

In other words

$$SE_{\text{difference}} = \sqrt{SE^2_{\text{first}} + SE^2_{\text{second}}}$$

Standard Error for Difference of Proportions

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$$SE_{\hat{p}_A - \hat{p}_B} = \sqrt{SE_{\hat{p}_A}^2 + SE_{\hat{p}_B}^2}$$

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Expanding this gives us

$$SE_{\hat{p}_A - \hat{p}_B} = \sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}$$

where n_A and n_B are the number of cases that $\hat{p}_A$ and $\hat{p}_B$ are based on. (You don't need to memorize these formulas!)

Normal Approximation of Sampling Distribution of $\hat{p}_A - \hat{p}_B$

- **When to use:** There are least 10 expected cases of each outcome in **both** groups

$$n_A p_A \geq 10 \qquad n_A(1-p_A) \geq 10$$
$$n_B p_B \geq 10 \qquad n_B(1-p_B) \geq 10$$

- **Mean of the Sampling Distribution of $\hat{p}_A - \hat{p}_B$:** Equal to the difference of pProportions $(p_A - p_B)$
- **SD of the Sampling Distribution of $\hat{p}_A - \hat{p}_B$ (i.e., the SE)**:

$$SE_{\hat{p}} = \sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}$$

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Standard Error of a Difference

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CI Summary

0. Check whether Normal approximation is **reasonable**
1. If so, use the **confidence level** and a **Standard Normal** to get z-**scores** of the endpoints
   - 95%: $z_{\text{endpoint}} = \pm 1.96 \ (\approx 2)$
   - 99%: $z_{\text{endpoint}} = \pm 2.58$
   - 90%: $z_{\text{endpoint}} = \pm 1.64$
2. Find the **mean** and **SD** of the estimated sampling distribution of interest
   - Mean = **Sample Statistic**
   - SD = **Standard Error**
3. **Convert** z scores **to the original scale** using the mean and standard deviation found in step 2.

$$\text{Endpoint} = \text{Sample Statistic} + z_{\text{endpoint}} \cdot \text{SE}$$

CI Summary: Difference of Proportions

0. Check whether Normal approximation is **reasonable** : All 4 of $n_A\hat{p}_A$, $n_A(1-\hat{p}_A)$, $n_B\hat{p}_B$ and $n_B(1-\hat{p}_B)$ are at least 10
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2. Find the **mean** and **SD** of the estimated sampling distribution of interest
   - Mean = **Sample Statistic** $= \hat{p}_A - \hat{p}_B$
   - SD = **Standard Error** $= \sqrt{\frac{\hat{p}_A(1-\hat{p}_A)}{n_A} + \frac{\hat{p}_B(1-\hat{p}_B)}{n_B}}$
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Penguin Breeding

- For the metal-band group, 39 of 122 (32%) penguin-seasons resulted in offspring
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Descriptive Statistics:

$$n_{\text{metal}} = _____ \qquad n_{\text{control}} = _____$$

$$\hat{p}_{\text{breed|metal}} = _____ \qquad \hat{p}_{\text{breed|control}} = _____$$

$$\hat{p}_{\text{breed|metal}} - \hat{p}_{\text{breed|control}} = _____$$

Penguins Breed Confidence (Interval)

Let's find a **90% CI for** $p_A - p_B$.

$$CI : \text{Sample Stat} \pm z_{\text{endpoint}} \cdot \widehat{\text{SE}}$$

0. **Check:** Is the Normal approximation reasonable?
1. **Find z-scores of Endpoints:** $z_{\text{endpoint}} =$ _____
2. **Sample Statistic and Standard Error:**

$$\text{Sample Stat} = \hat{p}_M - \hat{p}_C = \text{_____}$$

$$\widehat{\text{SE}} = \sqrt{\frac{\hat{p}_M(1-\hat{p}_M)}{n_M} + \frac{\hat{p}_C(1-\hat{p}_C)}{n_C}} = \text{_____}$$

3. **Convert to Units of Parameter:** $\text{Sample Stat} \pm z_{\text{endpoint}} \cdot \widehat{\text{SE}}$

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3. **Convert to Units of Parameter:** $\text{Sample Stat} \pm z_{\text{endpoint}} \cdot \widehat{\text{SE}}$

$$-0.12 \pm 1.64 \cdot 0.058 = -0.215 \text{ to } -0.025$$

Conclusion in Context:

We are **90% confident** that metal bands would lead to **between a 2.5 and 21.5 percentage point reduction** in the breeding rate

Outline

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Hypothesis Test for Difference of Proportions

Hypothesis Test: Difference of Two Proportions

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- So, its **mean** is **the null parameter**, with standard deviation based on the sample size and the null parameter

$$\widehat{SE}_{\hat{p}_A - \hat{p}_B} = \sqrt{\widehat{SE}^2_{\hat{p}_A} + \widehat{SE}^2_{\hat{p}_B}} = \sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}$$

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- What should we plug in for p_A and p_B, according to H_0?

Null Standard Error

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$$\widehat{SE}_{\hat{p}_A - \hat{p}_B} = \sqrt{\frac{p_{\text{both}}(1-p_{\text{both}})}{n_A} + \frac{p_{\text{both}}(1-p_{\text{both}})}{n_B}}$$

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- We can estimate p_{both} by **combining the groups** and calculating the **combined sProportion**
- Note: We held this proportion constant **already** when doing a randomization test.

Penguin Breeding

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What is $\hat{p}_{\text{breed}|\text{combined}}$?

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What is $\hat{p}_{\text{breed|combined}}$?

$$\hat{p}_{breed|combined} = \frac{39 + 70}{122 + 160} = \frac{109}{282} = 0.39$$

P-values Using Normal Approximation

0. Check whether the **approximation is reasonable**
1. If so, find the **mean and SD** of the Normal approximation of the **sampling distribution according to** H_0
   - Mean $=$ **Null Parameter Value**
   - SD $=$ $\widehat{\textbf{SE}}$
2. Find the z-**score corresponding to the sample statistic** within this Normal distribution (our **test statistic**)

$$z_{\text{obs}} = \frac{\text{Observed Sample Statistic} - \text{Null Parameter}}{\text{Standard Error}}$$

3. The P-value is the **proportion of values past** z (or past z and $-z$ if two-tailed) **in the Standard Normal**

P-values Using Normal Approximation : Difference of Proportions

0. Check whether the **approximation is reasonable**: All 4 of $n_A\hat{p}_{combined}$, $n_A(1-\hat{p}_{combined})$, $n_B\hat{p}_{combined}$ and $n_B(1-\hat{p}_{combined})$ are at least 10
1. If so, find the **mean and SD** of the Normal approximation of the **sampling distribution according to** H_0
   - Mean = **Null Parameter Value** $= p_0 = 0$
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Example: Penguins Again! But Different!

Descriptive Statistics:

$$n_{\text{metal}} = 122 \qquad n_{\text{control}} = 160$$

$$\hat{p}_{\text{breed|metal}} = 0.32 \qquad \hat{p}_{\text{breed|control}} = 0.44$$

$$\hat{p}_{\text{breed|metal}} - \hat{p}_{\text{breed|control}} = -0.12$$

$$\hat{p}_{\text{breed|combined}} = 0.39$$

0. **Check:** Is the Normal approximation reasonable?

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$$\hat{p}_{\text{breed|metal}} - \hat{p}_{\text{breed|control}} = -0.12$$
$$\hat{p}_{\text{breed|combined}} = 0.39$$

0. **Check:** Is the Normal approximation reasonable? **Yes**; the smallest of the four **expected counts** is $0.39 \cdot 122 = 47.6$

Do metal bands reduce breeding chances?

1. **Mean and SD of Normal Approximation:**
   (a) Mean is the **Null Parameter:** $p_M - p_C = \mathbf{0}$
   (b) SD is the **Standard Error:**

$$\widehat{SE} = \sqrt{\frac{\hat{p}_{\text{combined}}(1-\hat{p}_{\text{combined}})}{n_{\text{Metal}}} + \frac{\hat{p}_{\text{combined}}(1-\hat{p}_{\text{combined}})}{n_{\text{Control}}}}$$

$$= \underline{\qquad\qquad\qquad\qquad}$$

2. **Test Statistic:**

$$z_{\text{obs}} = \frac{\text{Observed Stat.} - \text{Null Param.}}{\widehat{\text{SE}}}$$

$$= \underline{\qquad\qquad\qquad\qquad}$$

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$$= \sqrt{\frac{0.39 \cdot 0.61}{122} + \frac{0.39 \cdot 0.61}{160}} = \mathbf{0.0586}$$

2. **Test Statistic:**

$$z_{\text{obs}} = \frac{\text{Observed Stat.} - \text{Null Param.}}{\widehat{\text{SE}}}$$

$$= _____$$

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2. **Test Statistic:**

$$z_{\text{obs}} = \frac{\text{Observed Stat.} - \text{Null Param.}}{\widehat{\text{SE}}}$$

$$= \frac{-0.12 - 0}{0.0586} = \mathbf{-2.048}$$

3. P-value and Conclusion in Context:

$$P\text{-value} = _____$$

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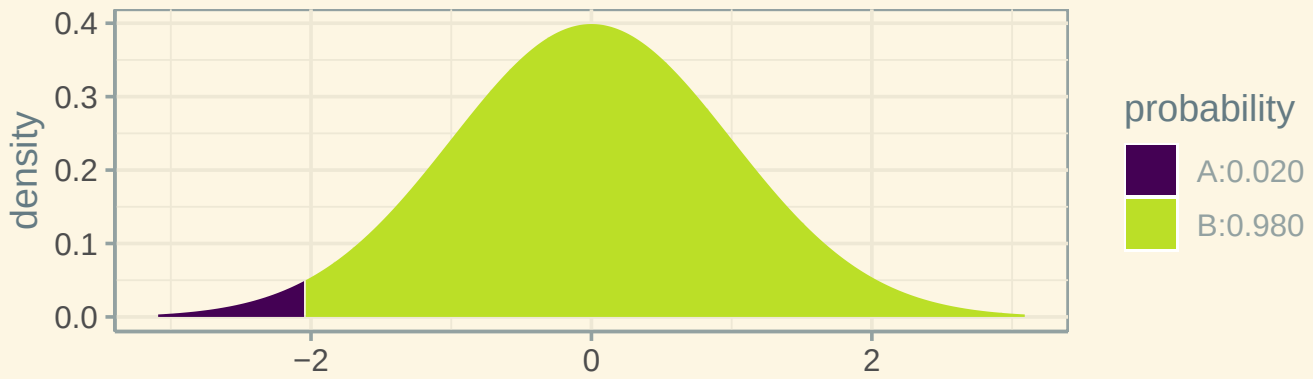
```
2 * pdist("norm", q = -2.048, lower.tail = TRUE)
```




```
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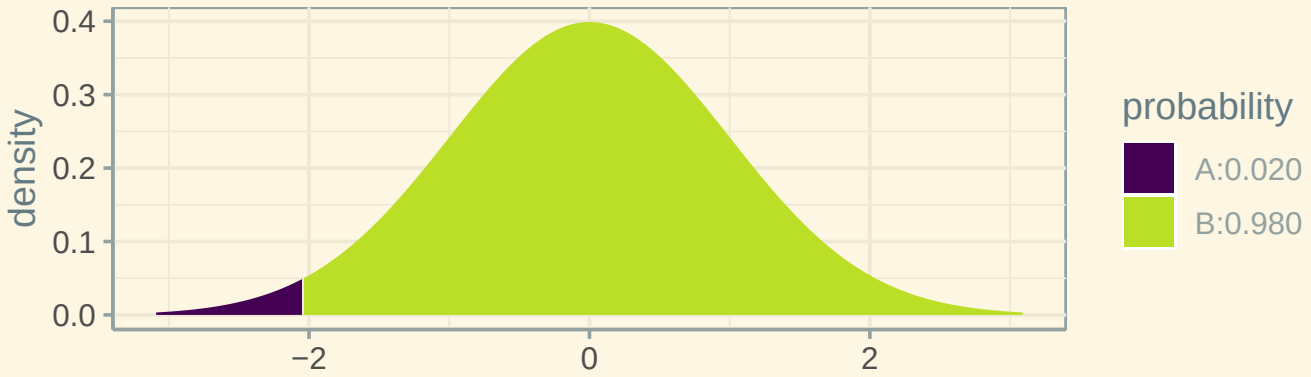


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2 * pdist("norm", q = -2.048, lower.tail = TRUE)
```

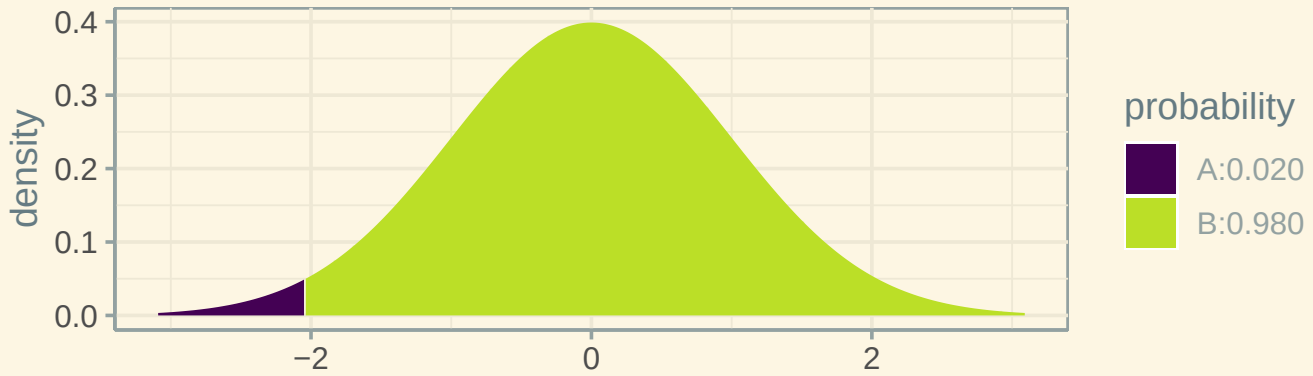


```
[1] 0.04056
```

Conclusion in Context:

We have **statistically significant evidence** ($z = -2.048$, $p = 0.041$ (two-tailed)) that **metal bands reduce breeding rates in penguins**