

STAT 113

Post-Break Review

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Inference

- Sampling Distributions

Confidence Intervals

- Big Picture

- Bootstrap CIs

- Precision

Hypothesis Testing

- Null and Alternative Hypotheses

- Randomization Distributions

- P-values

- Errors

Reminders and Announcements

- HW6 due
- Project Proposals due Friday
- Grade spreadsheets via Google Docs

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Variability due to Sampling

- Each sample differs from the population, so sample information is an imperfect reflection.
- However, sample statistics are “usually” (e.g., 95% of the time) “close” (e.g., within 2 SE) to the corresponding population parameter.
- This means two things
 - (a) We can estimate population parameters using sample statistics, but there will be a margin of error (confidence intervals)
 - (b) We can test claims about the population using a sample, by seeing how unlikely the sample statistic would be if the claim were false (hypothesis testing).

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Sampling Distributions

Sampling Distribution Definition

Consider all possible random samples of a fixed size, n from a population. Each one has its own value for a particular **statistic** (like \bar{x} , or \hat{p} , or r). A **sampling distribution** is the collection of all of those \bar{x} values (or whatever the statistic is)

Self-Check Quiz (Twenty Questions Plus Five)

1. The “cases” that make up a sampling distribution are _____ **samples of a fixed size**
2. (a) If we are interested in estimating or testing a hypothesis about a population mean, we should investigate the sampling distribution of what variable? **the corresponding sample mean**
(b) What if we are interested in a population proportion? **the corresponding sample proportion**
(c) A difference of population means? **the difference of sample means**
(d) A population correlation? **the sample correlation**
3. The **standard error** of the population parameter is the _____ **standard deviation** of the _____ **sampling distribution** of the sample **statistic**.

Properties of Sampling Distributions

Most (about 95%) of *simple random* samples have a sample mean (\bar{x}) which is within 2 Standard Errors of the population mean (μ). Therefore, about 95% of the time, the population mean will be within 2SE of the *sample* mean!

4. A similar statement holds for some other statistics/parameters, under a particular condition. What condition? **The sampling distribution needs to be (approximately) symmetric and bell-shaped**

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Confidence Intervals

Confidence Intervals

A *point estimate* of some population parameter (like a mean), together with some measure of our confidence/uncertainty (e.g., MoE), defines a **confidence interval**.

If we have the standard error and the sampling distribution is symmetric and bell-shaped, we get a 95% CI using

$$\text{point estimate} \pm 2SE$$

Confidence Interval Interpretation

5. Which of the following are valid interpretations of what confidence intervals mean?
- (a) We can be 93% confident that the population parameter falls in the 93% confidence interval.
 - (b) A 93% CI contains 93% of the cases in the population.
 - (c) A 93% CI contains 93% of the cases in the sample.
 - (d) 93% of 93% CIs contain the population parameter.
 - (e) 93% of samples have a statistic that falls in the 93% CI.

Confidence Interval Interpretation

5. Which of the following are valid interpretations of what confidence intervals mean?
- (a) We can be 93% confident that the population parameter falls in the 93% confidence interval.
 - (b) A 93% CI contains 93% of the cases in the population.
 - (c) A 93% CI contains 93% of the cases in the sample.
 - (d) 93% of 93% CIs contain the population parameter.
 - (e) 93% of samples have a statistic that falls in the 93% CI.

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Bootstrap Distributions

6. To construct a bootstrap distribution, we treat the _____ **sample** as the _____ **population**, and draw samples from it, being sure to _____ **replace the values** after each observation is drawn. We then compute the statistic of interest for each sample. The collection of these statistics form the bootstrap distribution.

Bootstrap Distributions

7. We use bootstrap distributions because they allow us to estimate _____the standard error and/or construct _____confidence intervals.
8. Bootstrap distributions are centered at the _____observed sample statistic.
9. The standard deviation of the bootstrap distribution can be used as an estimate of _____the standard error.
10. We can get the endpoints of a 94% confidence interval using a bootstrap distribution using the _____3rd percentile / 0.03 quantile and the _____97th percentile / 0.97 quantile of the distribution.

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Factors that Affect the Size of the Confidence Interval

11. Name two factors that affect the width of a confidence interval, and indicate whether there is a positive or negative relationship.

As the sample size increases the width of the CI decreases (a negative relationship). As the confidence level increases, the width of the CI increases (a positive relationship).

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Basic Logic

Null and Alternative Hypotheses

The **null hypothesis** (H_0) represents the “fallback”, or “skeptical” position (“nothing to see here”). The **alternative hypothesis** (H_1) represents “something interesting is happening”; usually what we suspect is true when we do the study.

12. Both H_0 and H_1 are statements about characteristics of _____ **the population**.

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Randomization Distributions

13. To test hypotheses, we construct **randomization distributions**. These represent hypothetical outcomes of a study, assuming that _____ H_0 (the null hypothesis) is true.
14. Randomization distributions are typically centered at _____ the population parameter value according to H_0 .
15. This stands in contrast to bootstrap distributions, which are typically centered at _____ the observed value of the sample statistic.

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P-values

16. The P -value represents the likelihood of getting _____ a value at least as extreme as the one observed, assuming that _____ H_0 is true.
17. We reject H_0 when the P -value is _____ small compared to the _____ significance level (α). When this happens we say the result is _____ statistically significant.
18. We can calculate the P -value by constructing a _____ randomization distribution and finding the proportion of _____ cases within it that lie at or beyond _____ the observed statistic.

P-values

19. If our H_1 is non-directional (two-tailed), then we also need to consider the cases that lie at or beyond _____ the “mirror image” of the observed statistic.
20. In this (two-tailed) case, how is the P -value obtained from these two proportions? By adding them together
21. Does rejecting H_0 with a two-tailed test require a bigger or smaller absolute difference between the sample statistic and the null parameter value than with a one-tailed test (all else equal)? It requires a bigger difference, because, for a given difference, the P -value is about twice as large, so the difference has to be larger to get the P -value below α . Equivalently: the rejection region uses a smaller part of each tail, so the difference has to be bigger to get out into that region.

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Making the Wrong Decision

22. Sometimes our rejection decision will be wrong. When we *incorrectly reject* H_0 , this is called a _____ **Type I Error** or a _____ **False Discovery**, and means that _____ we will mistakenly think there is an “effect” when there is none.
23. When we *incorrectly fail to reject* H_0 , this is called a _____ **Type II Error** or a _____ **Missed Discovery**, and means that _____ we will mistakenly think there is no “effect” when there is one.
24. Of the times that H_0 is true, the proportion of the time we will reject it (incorrectly) is determined by _____ α and is equal to _____ α .
25. We can make it easier or harder to reject H_0 by _____ **increasing** or _____ **decreasing** the _____ α level (respectively). What effect will this