

## STAT 113: TESTING FOR A DIFFERENCE OF PROPORTIONS

Are metal bands harmful to penguins (the kind used for tagging, not the kind that inspires head banging)? Researchers (Saraux et al., 2011) investigated this question with a sample of 20 penguins near Antarctica. All of these penguins had already been tagged with RFID (radio frequency identification) chips, and the researchers randomly assigned 10 of them to receive a metal band on their flippers in addition to the RFID chip. The other 10 penguins did not receive a metal band. Researchers then kept track of which penguins survived for the 4.5-year study and which did not.

- (1) Identify the explanatory and response variables and classify each as categorical or quantitative.
  
  
  
  
  
  
  
  
  
  
- (2) Why did the researchers include a comparison group in this study? Why not just see how many penguins survived while wearing a metal band?
  
  
  
  
  
  
  
  
  
  
- (3) Is this an observational study or an experiment? Explain how you know.
  
  
  
  
  
  
  
  
  
  
- (4) State the appropriate null and alternative hypotheses to be tested, both in words and with parameter symbols.

- (5) The researchers found that 9 of the 20 penguins survived, of whom 3 had a metal band and 6 did not. Organize these results into the following  $2 \times 2$  table.

	No metal band (control)	Metal band	Total
Survived			
Didn't survive			
Total	10	10	20

- (6) Calculate the proportion who survived in each group. Also calculate the difference between these proportions (subtracting the “metal band” proportion from the “control” proportion). Did the “metal band” group have a smaller proportion that survived than the control group in the sample?
- (7) Is it possible that this difference could have happened even if the metal band had no effect; i.e., simply due simply to the random nature of assigning penguins to groups (i.e., the luck of the draw)?
- (8) How unlikely is this to happen by chance? To answer this question we turn to simulation. As usual, we simulate outcomes assuming that the null hypothesis is true: that the metal band has no effect on penguin survival. More precisely, **we assume that the 9 penguins that survived would have done so with or without the metal band, and the 11 that did not would not have survived either way. The random part is which penguins ended up in which group (metal band or no metal band).** We can simulate this assignment with playing cards.

Select a group of cards to represent the penguins in this study. We only need to use the colors of the cards. How many cards do we need? What will each card represent? What should the colors represent? (Hint: the colors stay with the “penguins” across simulations.) How many cards will you use of each color?

- (9) For each simulated “experiment”, you will shuffle the cards and then randomly deal them into two stacks, representing the random part of the experiment. What will these stacks represent, and how many cards will you deal into each stack?

- (10) Perform this shuffling and dealing. Then fill in the table of simulated results below.

	No metal band (control)	Metal band	Total
Survived			
Didn't survive			
Total			

Difference in proportions who survived (control minus metal band):

Repeat this shuffling and dealing a second time. Again fill in the table of simulated results:

	No metal band (control)	Metal band	Total
Survived			
Didn't survive			
Total			

Difference in proportions who survived (control minus metal band):

- (11) Combine your results with the other groups by constructing a dotplot on the board of the differences in survival proportions (control minus metal band group).
- (12) Does it appear that the observed value of this statistic in the actual study would be unlikely to occur if there really were no genuine effect of the metal band? Explain how you can tell from the dotplot of class simulation results.

- (13) Based on the class simulation results, what is your preliminary conclusion about the effect of the metal band on penguins' survival? Explain the reasoning process behind this conclusion. Could it be wrong? How?
- (14) To get a good estimate of the  $P$ -value for this test, we can do thousands of simulations in StatKey. Go to <http://lock5stat.com/statkey/>, and select "Test for Differences in Proportions". Click "Edit Data" and enter the sample sizes and the counts that survived for each group (control and metal band). Be sure that the "Randomization method" selected is "Reallocation", and do 10,000 simulations of random assignment. What sample statistic is being plotted? Where is the null hypothesis randomization distribution centered?
- (15) Find the  $P$ -value corresponding to the data and hypotheses. State your conclusion both in terms of formal rejection or non-rejection of the null hypothesis, and in terms of the "real world" conclusion.

- (16) The actual study didn't look at just 20 penguins; it involved 100, with 50 randomly assigned to each group. In the full study, 16 of 50 penguins survived in the metal band group, and 31 of 50 survived in the control group. Find the observed difference in survival proportions. Simulate the null hypothesis with this larger study in StatKey, and find the  $P$ -value corresponding to this data. What does the  $P$ -value mean in context?
- (17) Based on the  $P$ -value from your simulation, do the observed results provide convincing evidence that metal bands have a harmful effect on penguins' survival? Explain your reasoning process as if to someone with no training in statistics.
- (18) Is drawing a cause-and-effect conclusion justified from this study? Explain why or why not.
- (19) Explain why it makes sense that the conclusion from the full study is so different from the earlier conclusion from the partial study, even though the difference in conditional proportions of success between the two groups are identical between the full and partial studies.